

## **General Disclaimer**

### **One or more of the Following Statements may affect this Document**

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.
- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.
- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.
- This document is paginated as submitted by the original source.
- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

AD616343

① 9'

MIE SCATTERING AND ABSORPTION CROSS SECTIONS  
FOR ABSORBING PARTICLES

by

Gilbert N. Plass  
Southwest Center for Advanced Studies  
Dallas, Texas

Submitted to Applied Optics  
May, 1965

COPY	OF	1st
HARD COPY	\$.	2.00
MICROFICHE	\$.	0.50

32p

This work was supported by the Air Force Cambridge Research Laboratories, Office of Aerospace Research contract AF19(628)-5039 and by the National Aeronautics and Space Administration contract NsG-269-62.

EVALUATION COPY

PROCESSING COPY

ARCHIVE COPY

DDC  
JUN 18 1965  
RECEIVED

TISIA E

# MIE SCATTERING AND ABSORPTION CROSS SECTIONS FOR ABSORBING PARTICLES

Gilbert N. Plass

## Abstract

The Mie scattering and absorption cross sections for spherical particles are calculated as a function of the complex index of refraction,  $n = n_1 - in_2$ . Curves of the efficiency factors for absorption, scattering, and extinction are given for  $n_1 = 1.01, 1.33, 1.5, 2$  and for  $n_2 = 10^{-4}, 10^{-2}, 10^{-1}, 1$ , and  $10$ . The influence of the absorption on the efficiency factors is discussed. The results are compared with previously proposed simplified expressions for these factors. The variation of the scattered intensity with angle as  $n_2$  is varied is illustrated in some typical cases. Finally the influence of  $n_2$  on the narrow resonances which occur for large values of  $n_1$  is discussed.

---

The author is with the Southwest Center for Advanced Studies, P. O. Box 30365, Dallas, Texas 75230.

Received

This work was supported by the Air Force Cambridge Research Laboratories, Office of Aerospace Research and by the National Aeronautics and Space Administration.

## I. Introduction

The interaction of electromagnetic radiation with absorbing spheres is a problem which arises in many different branches of science. Absorbing particles occur abundantly in planetary atmospheres where their interaction with ultraviolet, visible, and infrared radiation is of great importance. Astrophysicists are interested in the modification of light by interstellar particles. Numerous applications occur in chemistry when the size distribution and composition of mixtures containing small particles is studied. Van de Hulst's book<sup>1</sup> is now the classic reference which discusses the Mie theory in detail together with many of its applications.

Until the advent of modern high-speed computers, relatively few Mie theory calculations had been done for absorbing particles. Simplified equations were developed for a few limiting cases. The few calculations which were performed are reviewed by Van de Hulst<sup>1</sup>. Extensive computations for particular absorbing substances have been performed with the aid of high speed computers. These calculations have been made for carbon<sup>2</sup>, aluminum and magnesium oxide<sup>3,4</sup>, and particularly water droplets (the literature on this subject is reviewed by Deirmendjian et al.<sup>5,6</sup>).

It is difficult to obtain an understanding of the behavior of the cross sections for absorbing particles from these results which were calculated for the particular values of the complex index of refraction appropriate for these particular substances.



Aside from the general discussion in Van de Hulst's book<sup>1</sup> the only attempt at understanding the general behavior of these cross sections has been made by Deirmendjian et al.<sup>5</sup> Although these authors discuss very clearly many of the general characteristics of these cross sections, their illustrations are again given for only a few particular values of the index.

In the present study the Mie cross sections are calculated for a wide range of complex indices of refraction. The particular values chosen do not necessarily correspond to real substances, but rather are intended to illustrate the variation in the cross sections as the absorption by the particle varies over a wide range. These results are then compared with some of the limiting approximations which have been proposed.

## II. Absorption Cross Section

The complex index of refraction is the sole physical parameter which determines the absorption and scattering of an electromagnetic wave by a spherical particle with a given ratio of radius to wavelength. The Mie theory<sup>7</sup> has been extensively described in the literature<sup>1</sup>. Some time ago an efficient program for the calculation of Mie scattering and absorption cross sections was developed by Stull and Plass<sup>2</sup> for use on an electronic computer. The details of the method are given in that paper. More recently some valuable improvements in the original program were made by P. J. Wyatt.

The results of the exact calculations for the absorption cross section made by this method are given by the solid curves of Figs. 1-4 with  $n_1 = 1.01; 1.33; 1.5; 2.0$  (where the index of refraction is written  $n = n_1 - in_2$ ) and for a range of  $n_2$  from zero to 10. The efficiency factor for absorption,  $Q_{abs}$ , (absorption cross section divided by cross-sectional area of particle) is plotted as a function of  $x = 2\pi a/\lambda$ , where  $a$  is the radius of the particle and  $\lambda$  is the wavelength.

One of the first features which one observed from these curves (and from further calculations which could not be shown here) is that  $Q_{abs} = (\text{constant})n_2$  for a fixed value of  $x$  and  $n_1$ , provided only that  $n_2 \leq 0.01$ . Thus, if a curve of  $Q_{abs}$  as a function of  $x$  is calculated for a particular  $n_1$  and for one value of  $n_2 < 0.01$ , then  $Q_{abs}$  is known immediately for all other values of  $n_2$  which satisfy the inequality. A linear dependence of  $Q_{abs}$  on  $n_2$  is to be expected, of course, when the absorption by the particle is sufficiently small.

The curves for  $Q_{abs}$  vary smoothly with  $x$  for  $n_1 = 1.01$  and 1.33, but for larger values of  $n_1$  some resonances appear. The qualitative appearance of the curves is the same in each of these figures, except for these minor resonances. For  $n_2 \leq 0.1$  the curves gradually approach their limiting values for large  $x$  from below. The curves for  $n_2 = 1$  and 10 reach a maximum value (the maximum value of  $Q_{abs}$  in all cases reported here is near 1.8 for  $n_2 = 1$ ) and approach their limiting values for large  $x$  from above. In almost all cases  $Q_{abs}$  increases slightly as  $n_1$  increases while  $n_2$  and  $x$  are held fixed.

An approximate equation for  $Q_{abs}$  was proposed by Van de Hulst<sup>1</sup>,

$$Q_{abs} = 2K(4xn_2) \quad , \quad (1)$$

where the function  $K$  is

$$K(w) = (1/2) + w^{-1}e^{-w} + w^{-2}(e^{-w}-1) \quad . \quad (2)$$

This equation is expected to be useful when both  $n_1 - 1 \ll 1$  and  $n_2 \ll 1$ .

For comparison purposes Eq. (1) is plotted as a dashed curve in Figs. 1-3. For  $n_1 = 1.01$ , the two curves coincide for  $n_2 \leq 10^{-2}$  on the scale of Fig. 1. There are relatively small differences between the curves for  $n_2 = 10^{-1}$ . For  $n_2 = 1$ , the approximate curve fails to reproduce the maximum and becomes more and more inexact as  $x$  becomes larger. The approximate equation fails entirely for  $n_2 = 10$ . When  $n_1 = 1.33$  or 1.5, the discrepancies between the two curves are fairly large even for small values of  $n_2$ , as is shown in Figs. 2 and 3. It should be noted that Eq. (1) is independent of  $n_1$  and thus does not reproduce the slow increase of  $Q_{abs}$  with  $n_1$  for a fixed  $x$  and  $n_2$ . For this reason Eq. (1) is not plotted in Fig. 4, as it is no longer a reasonable approximation to the exact curve.

For most values of  $x$ , the maximum value of  $Q_{abs}$  occurs for  $n_2$  in the range from 0.1 to 1. The value of  $Q_{abs}$  is an order of magnitude less when  $n_2$  increases to 10. The reason



for this behavior is that, when  $n_2 = 10$ , over 90% of the incident radiation is reflected, whereas only about 20% is reflected when  $n_2 = 1$  for the values of  $n_1$  considered here.

### III. Scattering Cross Sections

The efficiency factors for scattering are shown in Figs. 5-8 for  $n_1 = 1.01, 1.33, 1.5, 2.0$ . For small values of  $Q_{sca}$  before the first maximum in the curve,  $Q_{sca}$  increases approximately as  $x^2$ , whereas  $Q_{abs}$  increases as  $x$ .

The values of  $Q_{sca}$  are the same on the scale of these figures for all  $n_2 \leq 10^{-4}$  (actually the values do not vary by more than 2 units in the third significant figure over this entire range).

The influence of  $n_2$  on  $Q_{sca}$  is rather complicated. In Fig. 5 it is seen that, when  $x < 30$  and  $n_1 = 1.01$ ,  $Q_{sca}$  increases as  $n_2$  increases from zero to 10. Because of the small value of  $n_1$ , the first resonance lies beyond  $x = 30$  for small  $n_2$ . However, for  $n_2 = 1$  and 10, the maximum of the curve occurs at  $x$  values of about 3 and 1.3 respectively.

The curves in Figs. 6-8 show a number of resonances. The value of  $Q_{sca}$  always decreases at first as  $n_2$  increases whenever  $x$  is greater than about 2. When  $n_2 = 10^{-2}$ , the second maximum of the curve is appreciably lower than when  $n_2 \leq 10^{-4}$ . The succeeding waves in the curve are slowly damped out for  $n_2 = 10^{-2}$ . When  $n_2 = 0.1$ , only a few maxima can be seen in the curve before the waves are completely damped out. When  $n_2 = 1$  or 10 the curve has only one maximum and then decreases very slowly to its limiting value. Parts of these curves are above the

curves for smaller  $n_2$  values. For the range of  $n_1$  shown here the limiting value of  $Q_{sca}$  as  $x$  becomes very large is between 1.00 and 1.11 when  $n_2 \leq 0.1$ , is about 1.2 when  $n_2 = 1$ , and is near 1.95 when  $n_2 = 10$ .

#### IV. Extinctions Cross Section

The extinction efficiency factors for  $n_1 = 1.01, 1.33, 1.5, 2.0$  are shown in Figs. 9-12. Although this factor is merely the sum of the absorption and scattering efficiency factors, it seems worthwhile to present these curves because (1) curves on a linear rather logarithmic scale present different information; (2) the absorption and scattering efficiency factors make varying contributions to the whole as the parameters change; (3) a comparison with Van der Hulst's simplified equation is instructive.

The approximate equation for  $Q_{ext}$  proposed by Van der Hulst<sup>1</sup> is

$$Q_{ext} = 2 - 4 \exp(-\rho \tan \beta) (\rho^{-1} \cos \beta) \sin(\rho - \beta) - 4 \exp(-\rho \tan \beta) (\rho^{-1} \cos \beta)^2 \cos(\rho - 2\beta) + 4(\rho^{-1} \cos \beta)^2 \cos 2\beta, \quad (3)$$

where

$$\rho = 2x(n_1 - 1) \quad , \quad (4)$$

$$\tan \beta = n_2 (n_1 - 1)^{-1} \quad (5)$$

This equation is valid under the same conditions as Eq. (1). Equation (3) is plotted as a dashed line in Figs. 9-11.



Again the approximate and true curves are indistinguishable when  $n_1 = 1.01$  and  $n_2 \leq 10^{-2}$ . There are slight differences when  $n_2 = 10^{-1}$ . As expected Eq. (3) breaks down when  $n_2 \geq 1$  and does not reproduce the true curve.

When  $n_1 = 1.33$  and  $1.5$ , Eq. (3) reproduces the qualitative shape of the curves for  $n_2 \leq 10^{-1}$ . Nevertheless the actual values of the maxima and minima are poorly reproduced. Deirmendjian<sup>8</sup> has proposed an empirical formula which corrects this deficiency in Eq. (3). However, we have not compared it in detail with our results because of its complexity and because it does not appear to increase appreciably the range of validity of Eq. (3).

The curves for very small values of  $n_2$  do not attempt to represent the many small ripples which exist in these curves<sup>1</sup>. These are rapidly damped out as  $n_2$  increases. As  $x$  increases,  $Q_{\text{ext}}$  always approaches the value 2. As  $n_1$  increases, the first resonance in the curve becomes larger and narrower and occurs at smaller  $x$  values. It is virtually completely damped out when  $n_2 = 1$ .

The curve for  $n_2 = 10^{-1}$  in Fig. 12 illustrates how the resonances can be damped out as  $x$  increases. For  $x < 15$  this curve has maxima and minima at the same positions as the curve for smaller  $n_2$  values. However, as  $x$  becomes greater than 15 an elementary calculation shows that there is no longer any appreciable transmitted energy through the particle; all of the radiation which is not reflected is absorbed by the particle.



Thus this curve stops oscillating and decreases slowly in value as  $x$  increases further. The same effect can be seen in Figs. 10 and 11.

#### V. Angular Scattering

It is obviously impossible to show more than a few of the angular scattering patterns which were generated in this study. An illustrative sampling of some typical patterns is shown in Figs. 13-15. These figures are all drawn for the value  $x = 5$  and for the values  $n_1 = 1.33, 1.5$  and  $2$  and  $n_2$  from  $0$  to  $10$ . The solid curves are for the intensity  $i_1$  and the dashed curves for  $i_2$  (as defined by Van de Hulst<sup>1</sup>). The values were calculated every  $5^\circ$  and occasionally more closely when the curve varied rapidly.

The scattering diagrams for each of these values of  $n_1$  exhibit the same pattern of change as  $n_2$  increases. There are only minor variations in the shape of the curves as long as  $n_2 \leq 10^{-2}$ . When  $n_2 = 0.1$ , the minima in the curve for  $i_1$  either become more prominent and often sharper ( $n_1 = 1.33$  and  $1.5$ ) or tend to disappear and become broader ( $n_1 = 1.5$  and  $2$ ). On the other hand  $i_2$  seems always to develop deeper minima which become more pronounced as  $n_2$  increases to the value  $10$ . The minima of  $i_1$  nearly disappear as  $n_2$  increases to the value  $10$ . The scattering diagrams are most sensitive to the value of  $n_1$  when  $n_2$  is small and are nearly independent of  $n_1$  when  $n_2 = 10$ .



The values of the intensity at  $0^\circ$  and  $180^\circ$  are marked on the curve. It is interesting to note that the ratio of backward to forward scattering always decreases at first as  $n_2$  increases. The ratio is a minimum when  $n_2$  is between  $10^{-2}$  and 1 and then increases again for larger values of  $n_2$ .

#### VI. Resonances

As  $n_1$  becomes larger more and more resonances occur below a particular, but sufficiently large value of  $x$ . The effect on these resonances of absorption by the particle is illustrated in Fig. 16. When  $n_1 = 50$  the first resonance due to the magnetic dipole term occurs at  $x = 0.0627$ . Both the efficiency factors for scattering and absorption have a sharp maximum at this value (unless  $n_2 \gg 1$ ).

The values of  $Q_{\text{abs}}$  are proportional to  $n_2$  when  $n_2 \leq 10^{-2}$ . The same relationship holds for  $n_2 = 0.1$  and 1 provided in addition that  $Q_{\text{abs}} < 0.1$ . The maximum value of  $Q_{\text{abs}}$  at the resonance peak occurs when  $n_2 = 0.1$  and decreases for larger values of  $n_2$ . The half-width of the resonance is constant for any  $n_2 \leq 0.1$ , but increases as  $n_2$  becomes larger than 0.1. The resonance is relatively very broad for  $n_2 = 10$  with the half-width several orders of magnitude larger than for values of  $n_2 \leq 0.1$ .

The resonance in  $Q_{\text{sca}}$  occurs even if there is no absorption by the particle. The curves for different  $n_2$  values are identical on the scale of Fig. 16 as long as  $n_2 \leq 10^{-2}$ . The resonance

is so narrow that the maximum value can not be computed accurately. The value of  $Q_{sca}$  anywhere near the resonance decreases uniformly as  $n_2$  increases. The resonance has broadened appreciably when  $n_2 = 1$  and can no longer be seen in the curve for  $n_2 = 10$ .

REFERENCES

1. H. C. Van de Hulst, Light Scattering by Small Particles (Wiley, New York, 1957).
2. V. R. Stull and G. N. Plass, J. Opt. Soc. Am. 50, 121 (1960).
3. G. N. Plass, Appl. Opt. 3, 867 (1964).
4. G. N. Plass, Appl. Opt. (submitted for publication).
5. D. Deirmendjian, R. Clasen, and W. Viezee, J. Opt. Soc. Am. 51, 620 (1961).
6. D. Deirmendjian, Appl. Opt. 3, 187 (1964).
7. G. Mie, Ann. Physik 25, 377 (1908).
8. D. Deirmendjian, Quart. J. Roy. Meteorological Soc., 86, 371 (1960).

# LEGENDS FOR FIGURES

Fig. 1 Efficiency factor for absorption as a function of  $x = 2\pi a/\lambda$ , where  $a$  is the particle radius and  $\lambda$  is the wavelength for  $n_1 = 1.01$ . Curves are shown for  $n_2 = 10^{-4}, 10^{-2}, 10^{-1}, 1, 10$ . The solid curves are the exact results and the dashed curves are the approximate Eq. (1). These curves coincide on this scale for  $n_2 = 10^{-4}$  and  $10^{-2}$ .

Fig. 2 Efficiency factor for absorption as a function of  $x = 2\pi a/\lambda$  for  $n_1 = 1.33$ . The solid curves are the exact results and the dashed curves are the approximate Eq. (1).

Fig. 3 Efficiency factor for absorption as a function of  $x = 2\pi a/\lambda$  for  $n_1 = 1.5$ . The solid curves are the exact results and the dashed curves are the approximate Eq. (1).

Fig. 4 Efficiency factor for absorption as a function of  $x = 2\pi a/\lambda$  for  $n_1 = 2$ .

Fig. 5 Efficiency factor for scattering as a function of  $x = 2\pi a/\lambda$  for  $n_1 = 1.01$ .

Fig. 6 Efficiency factor for scattering as a function of  $x = 2\pi a/\lambda$  for  $n_1 = 1.33$ .

Fig. 7 Efficiency factor for scattering as a function of  $x = 2\pi a/\lambda$  for  $n_1 = 1.5$ .



Fig. 8 Efficiency factor for scattering as a function of  $x = 2\pi a/\lambda$  for  $n_1 = 2$ .

Fig. 9 Efficiency factor for extinction as a function of  $x = 2\pi a/\lambda$  for  $n_1 = 1.01$ . The solid curves are the exact results and the dashed curves are the approximate Eq. (3). These curves coincide on this scale for  $n_2 = 10^{-4}$  and  $10^{-2}$ .

Fig. 10 Efficiency factor for extinction as a function of  $x = 2\pi a/\lambda$  for  $n_1 = 1.33$ . The solid curves (use left-hand scale) are the exact results and the dashed curves (use right-hand scale) are the approximate Eq. (3).

Fig. 11 Efficiency factor for extinction as a function of  $x = 2\pi a/\lambda$  for  $n_1 = 1.5$ . The solid curves (use left-hand scale) are the exact results and the dashed curves (use right-hand scale) are the approximate Eq. (3).

Fig. 12 Efficiency factor for extinction as a function of  $x = 2\pi a/\lambda$  for  $n_1 = 2$ .

Fig. 13 Scattered intensity as a function of scattering angle for  $n_1 = 1.33$  and  $x = 2\pi a/\lambda = 5$ . The solid curve is the intensity  $i_1$  (as defined by Van de Hulst<sup>1</sup>) and the dashed curve is the intensity  $i_2$ . The logarithm of the intensity (1 division = a factor 10) is plotted against the scattering angle. The values of the intensity at  $0^\circ$  and  $180^\circ$  are indicated above the curves near the margin.



Fig. 14 Scattered intensity as a function of scattering angle for  $n_1 = 1.5$  and  $x = 2\pi a/\lambda = 5$ . See caption for Fig. 13.

Fig. 15 Scattered intensity as a function of scattering angle for  $n_1 = 2$  and  $x = 2\pi a/\lambda = 5$ . See caption for Fig. 13.

Fig. 16 Efficiency factors for both absorption and scattering as a function of  $x = 2\pi a/\lambda$  for  $n_1 = 50$  and various values of  $n_2$ . The curves for scattering coincide on this scale for  $n_2 \leq 10^{-2}$ .

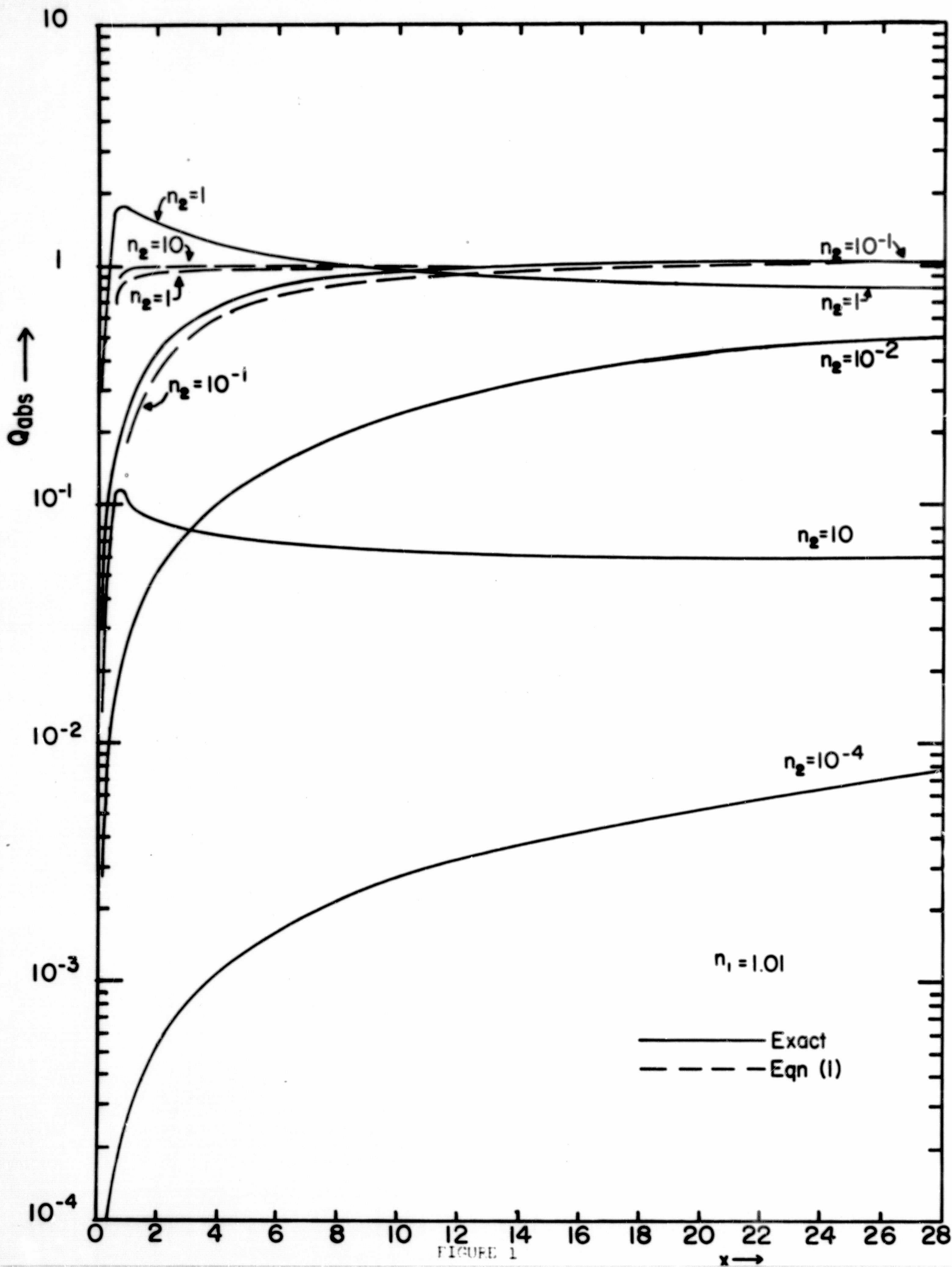


FIGURE 1

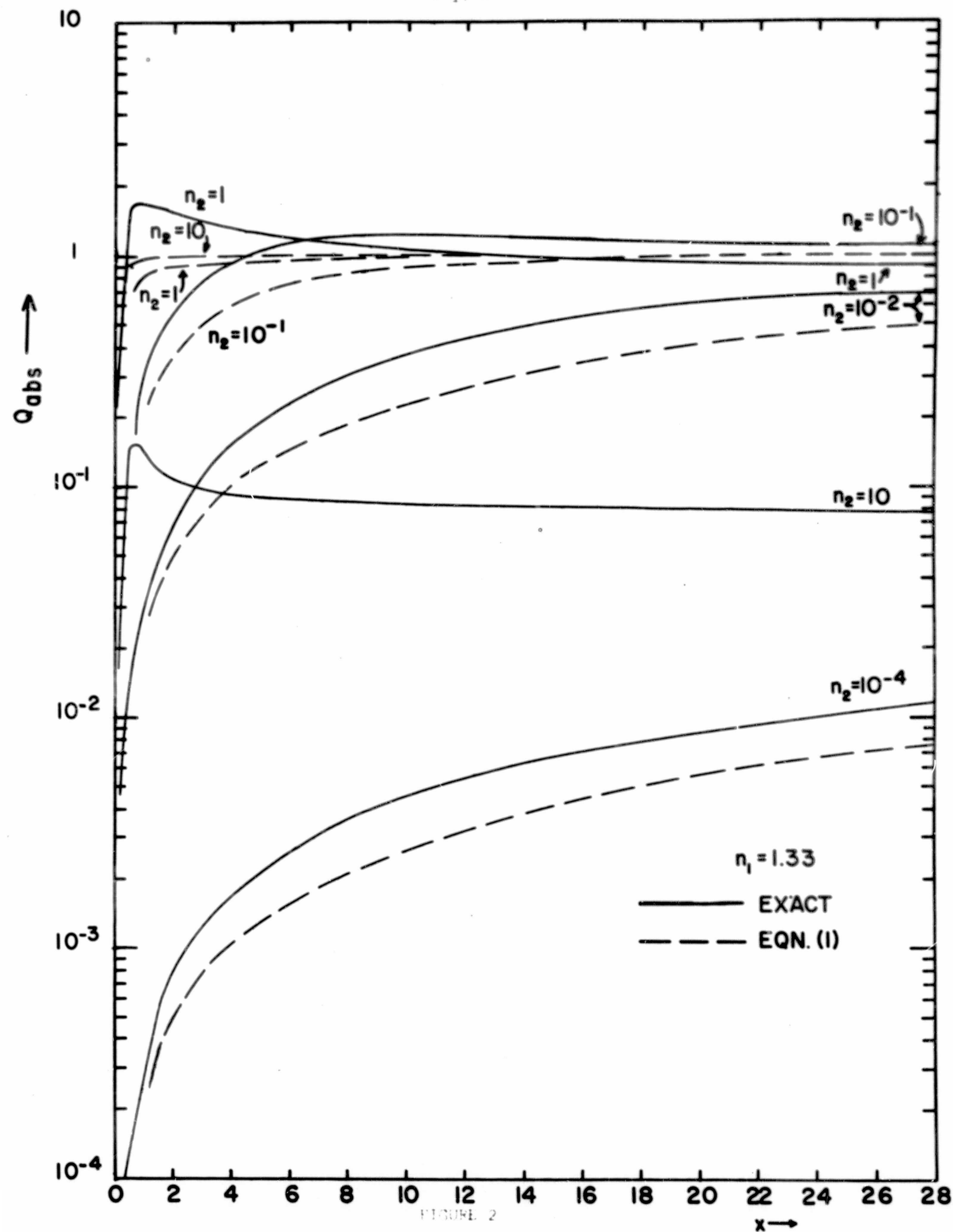


FIGURE 2

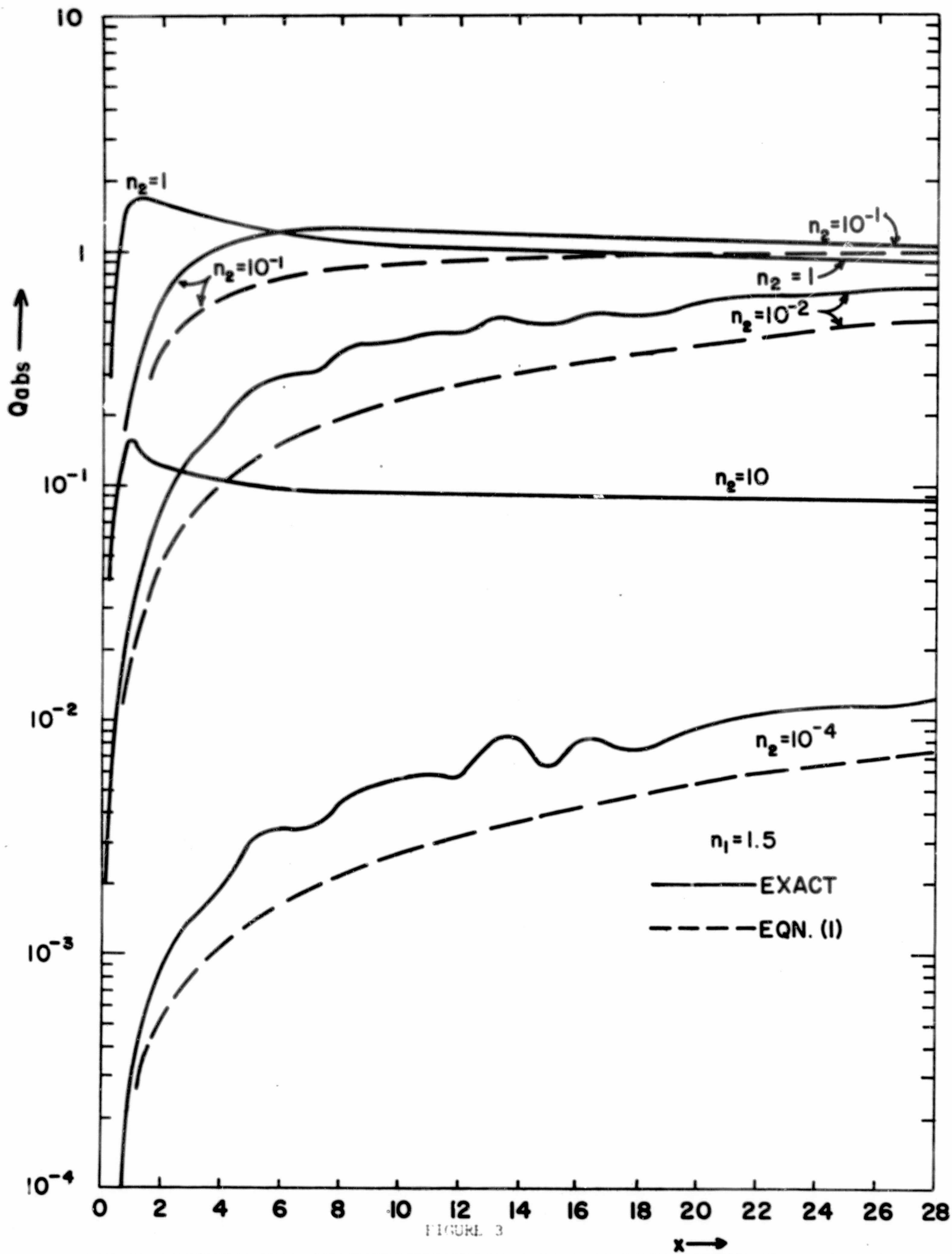
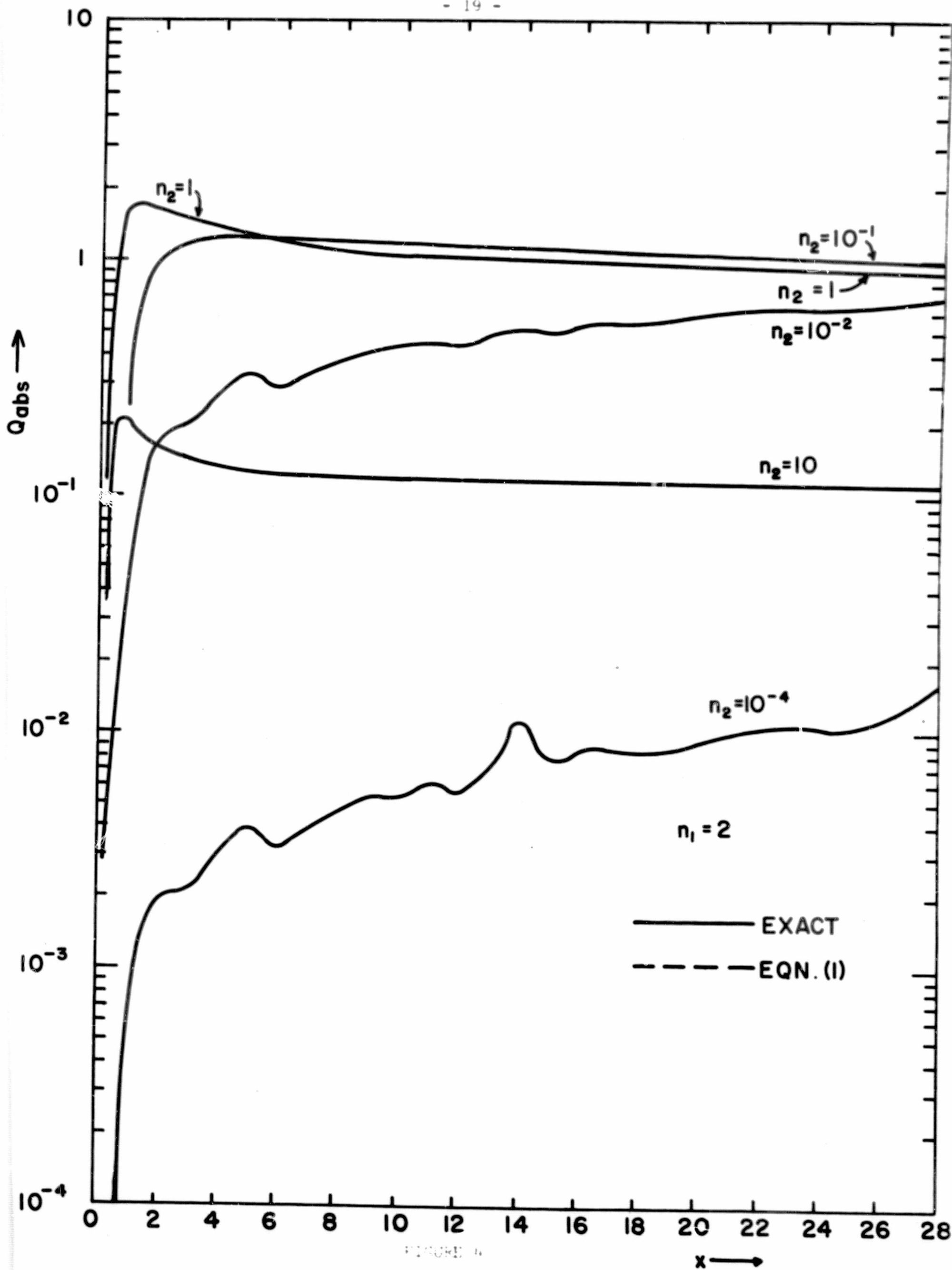
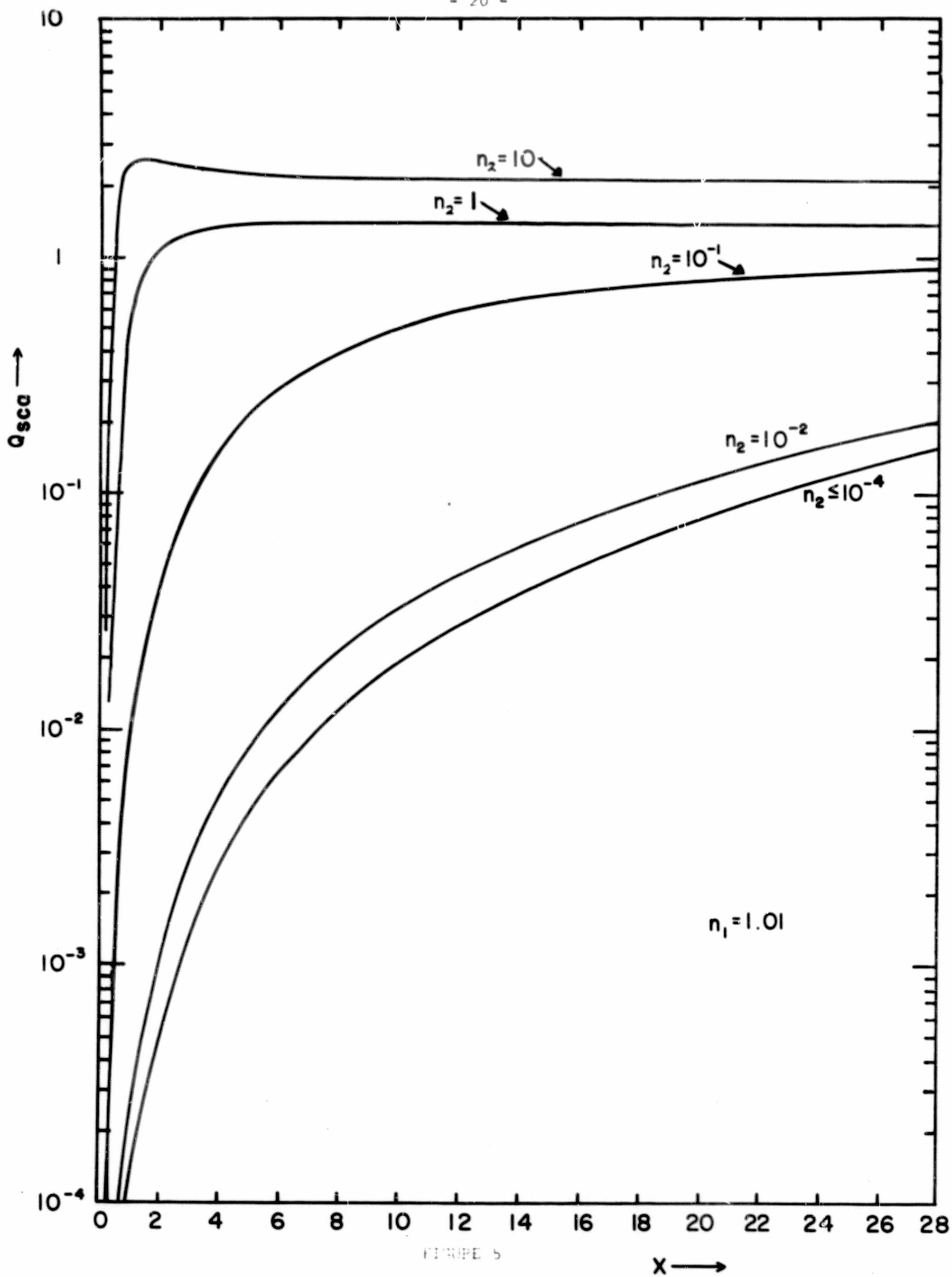


FIGURE 3







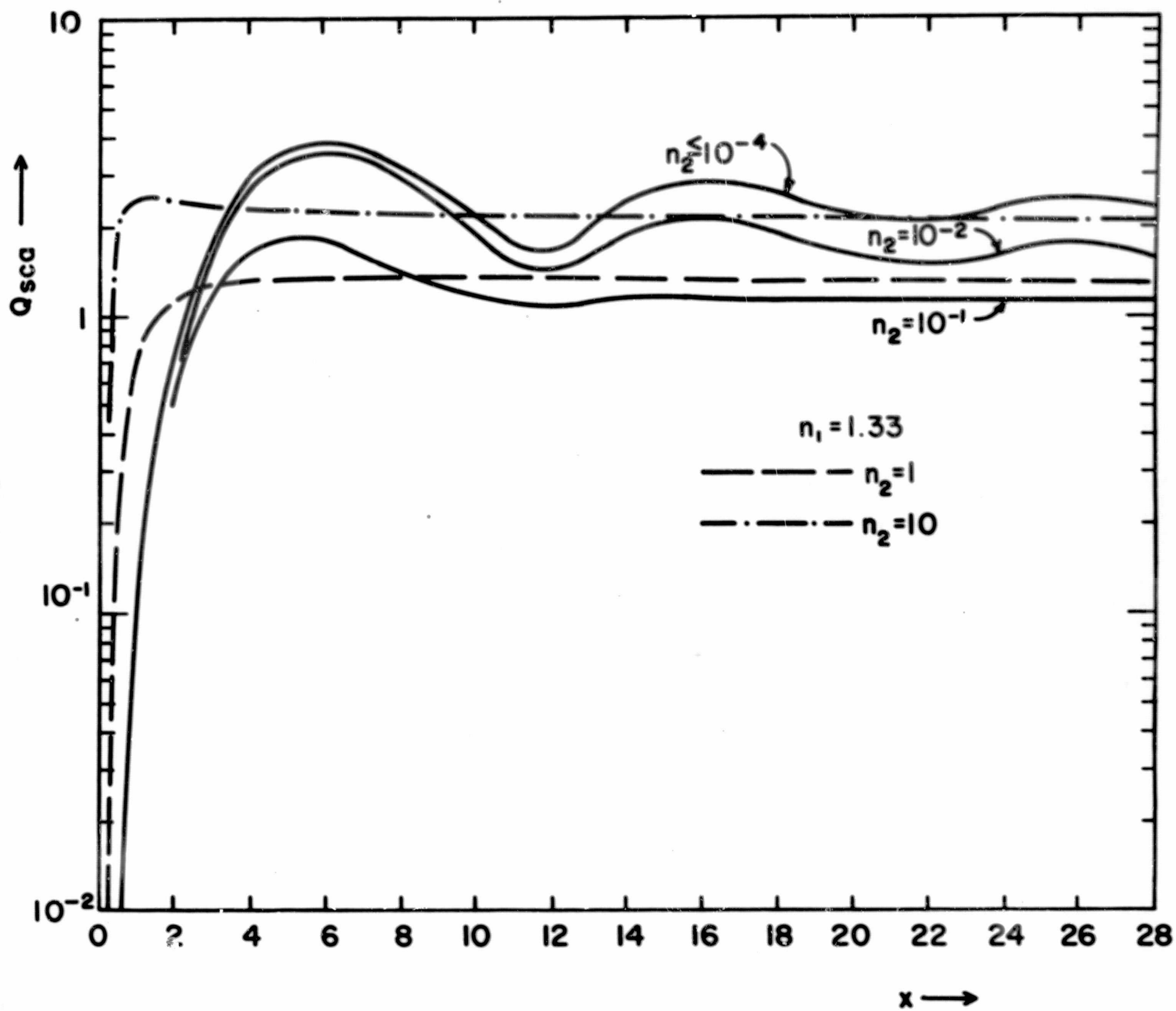


FIGURE 6

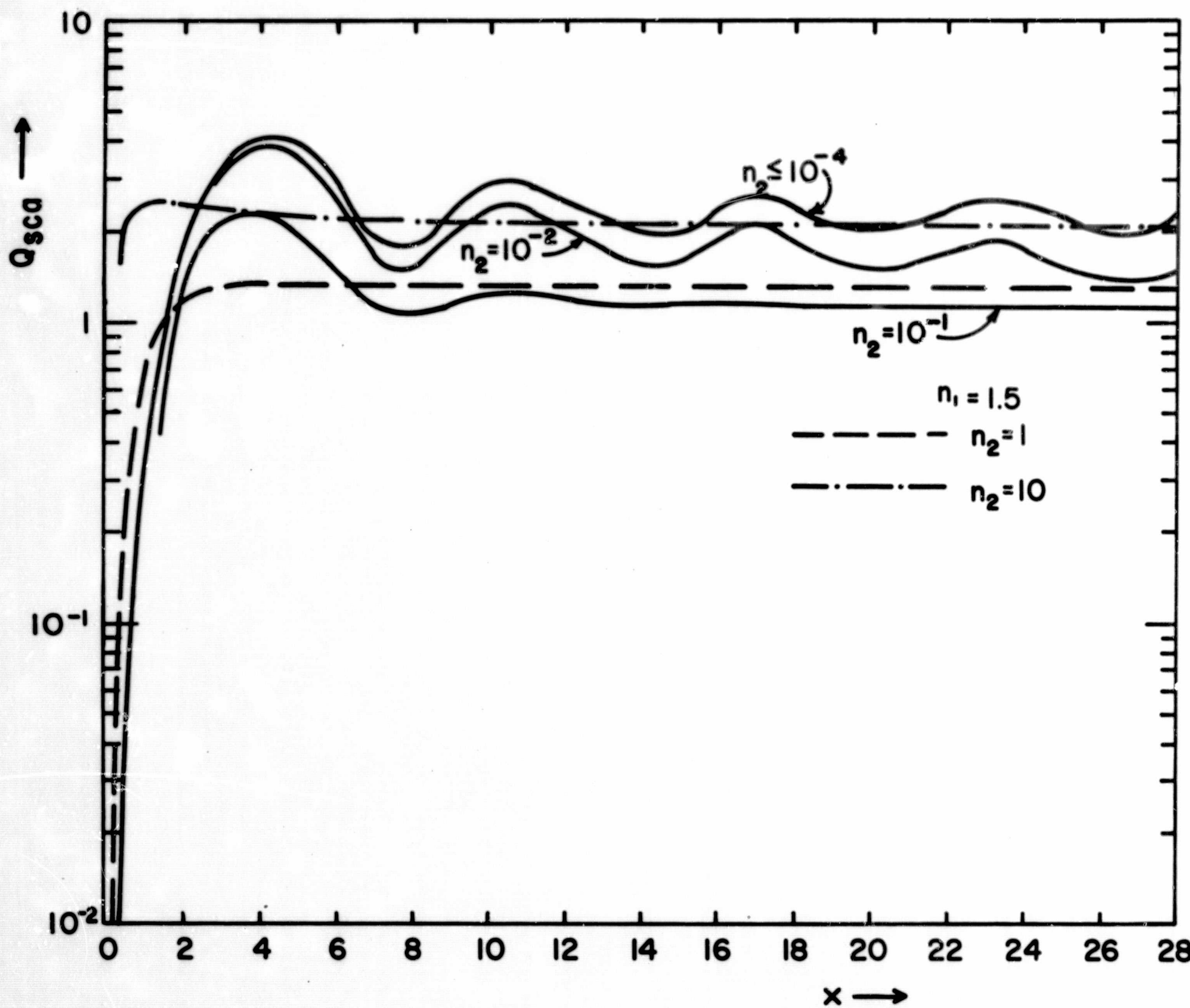


FIGURE 7

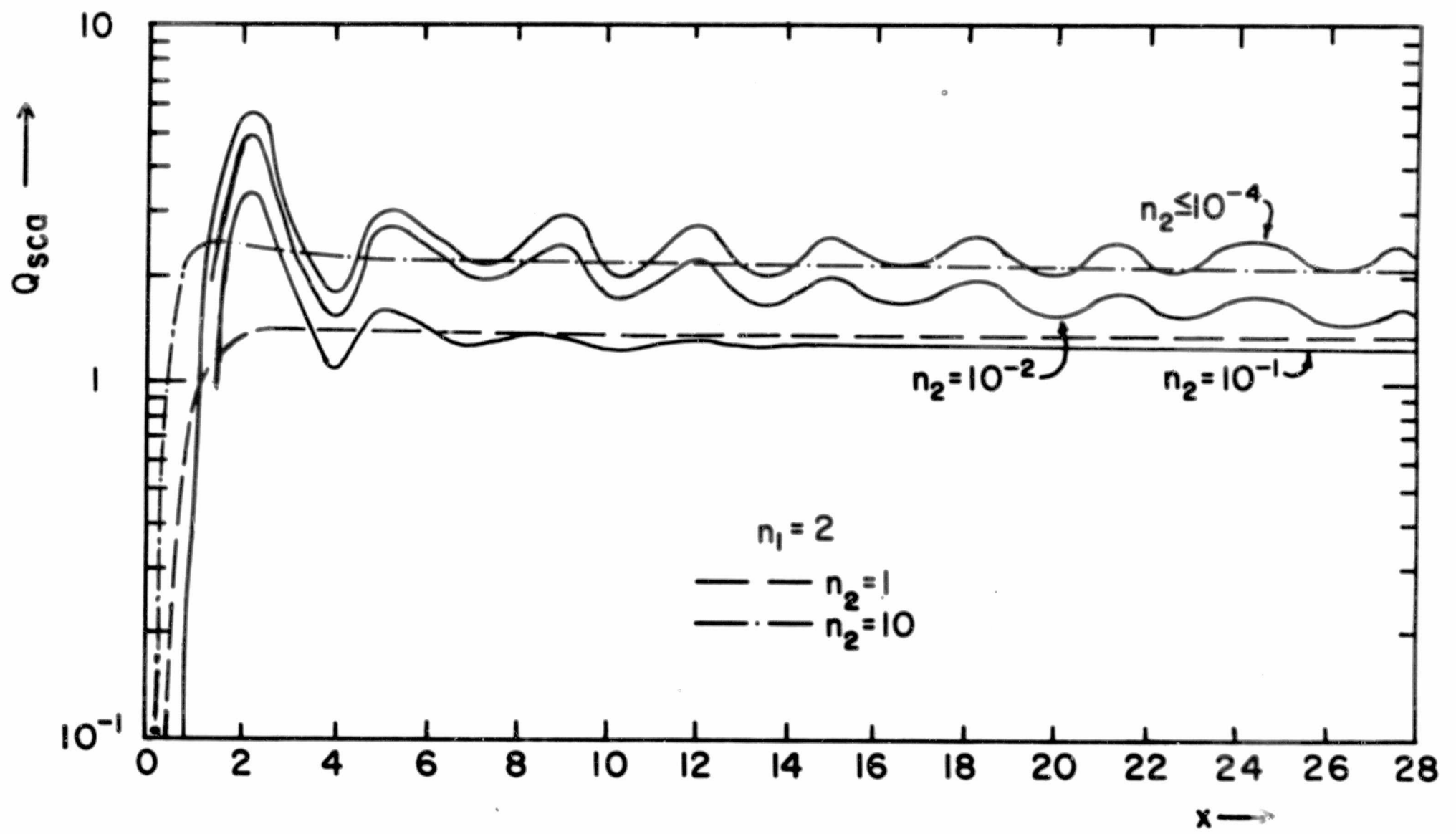


FIGURE 8

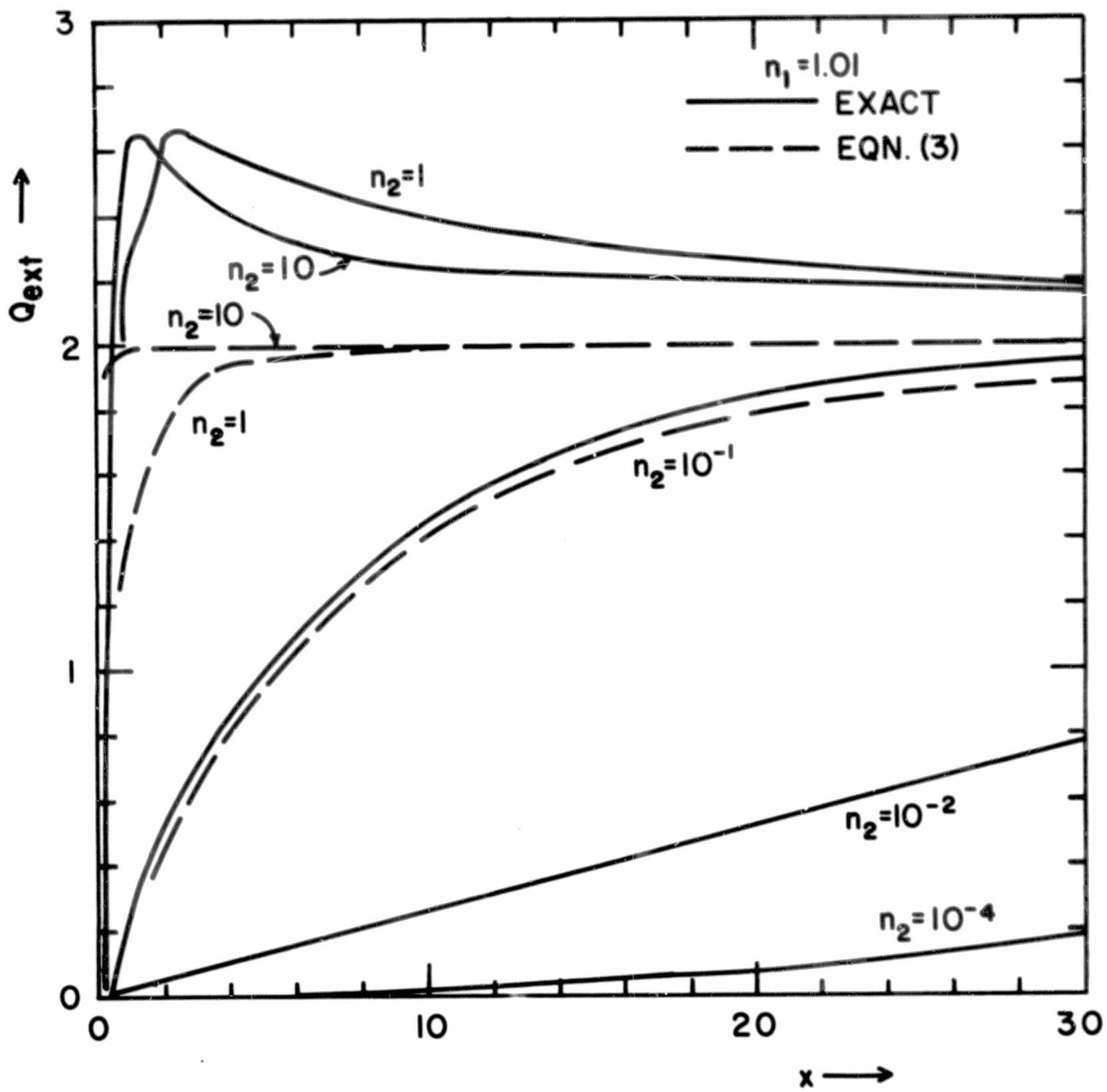


FIGURE 9

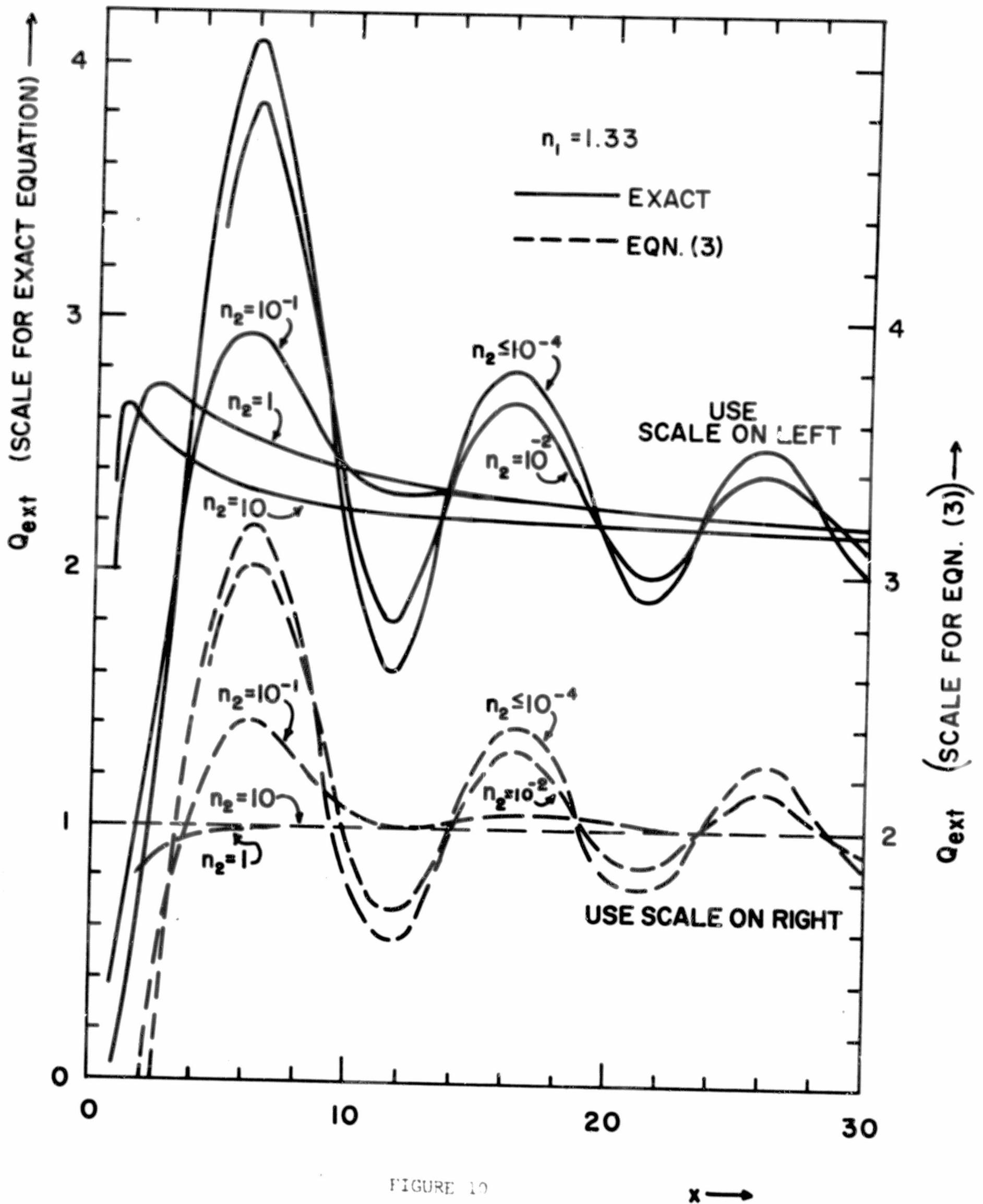


FIGURE 10

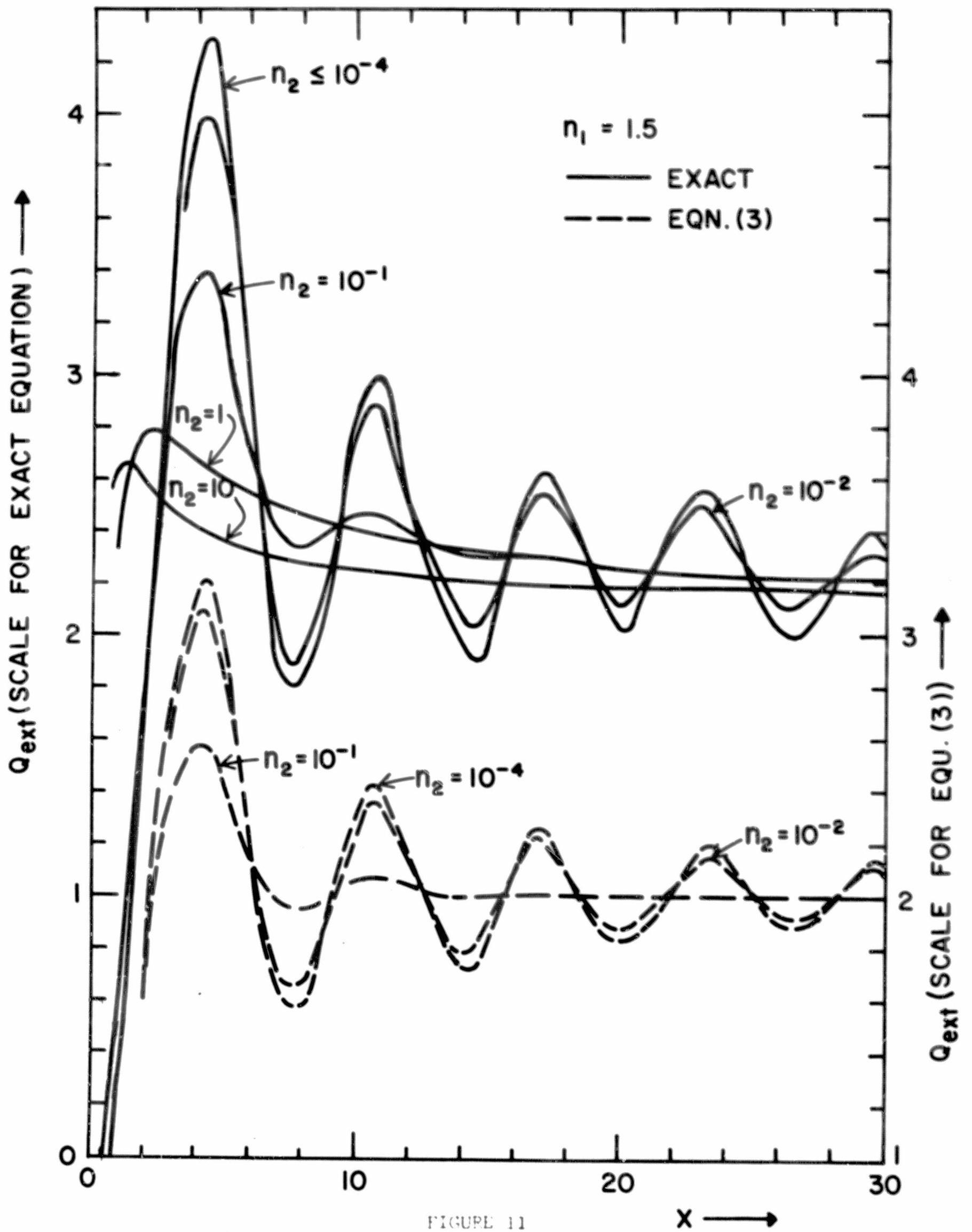


FIGURE 11



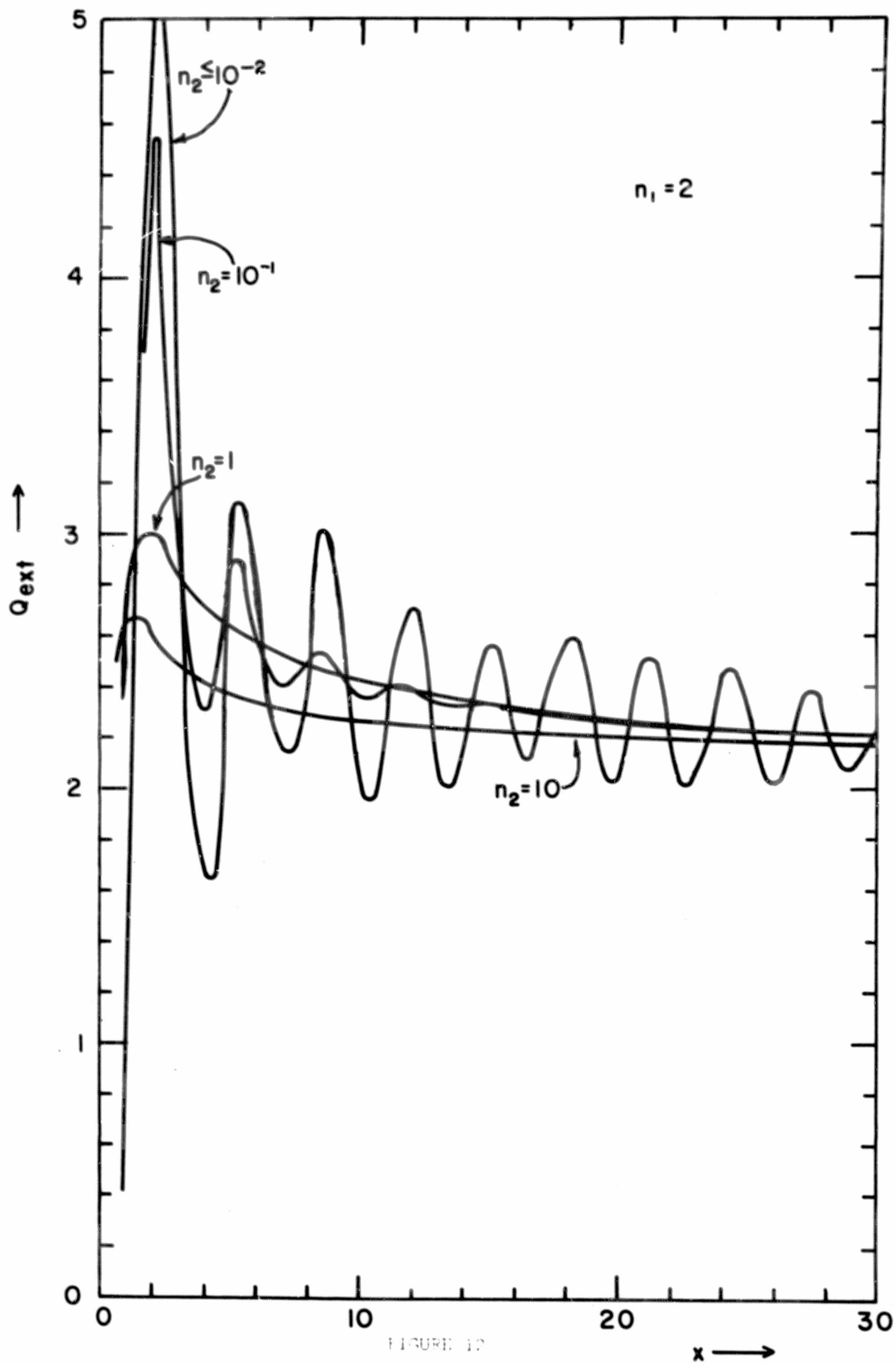
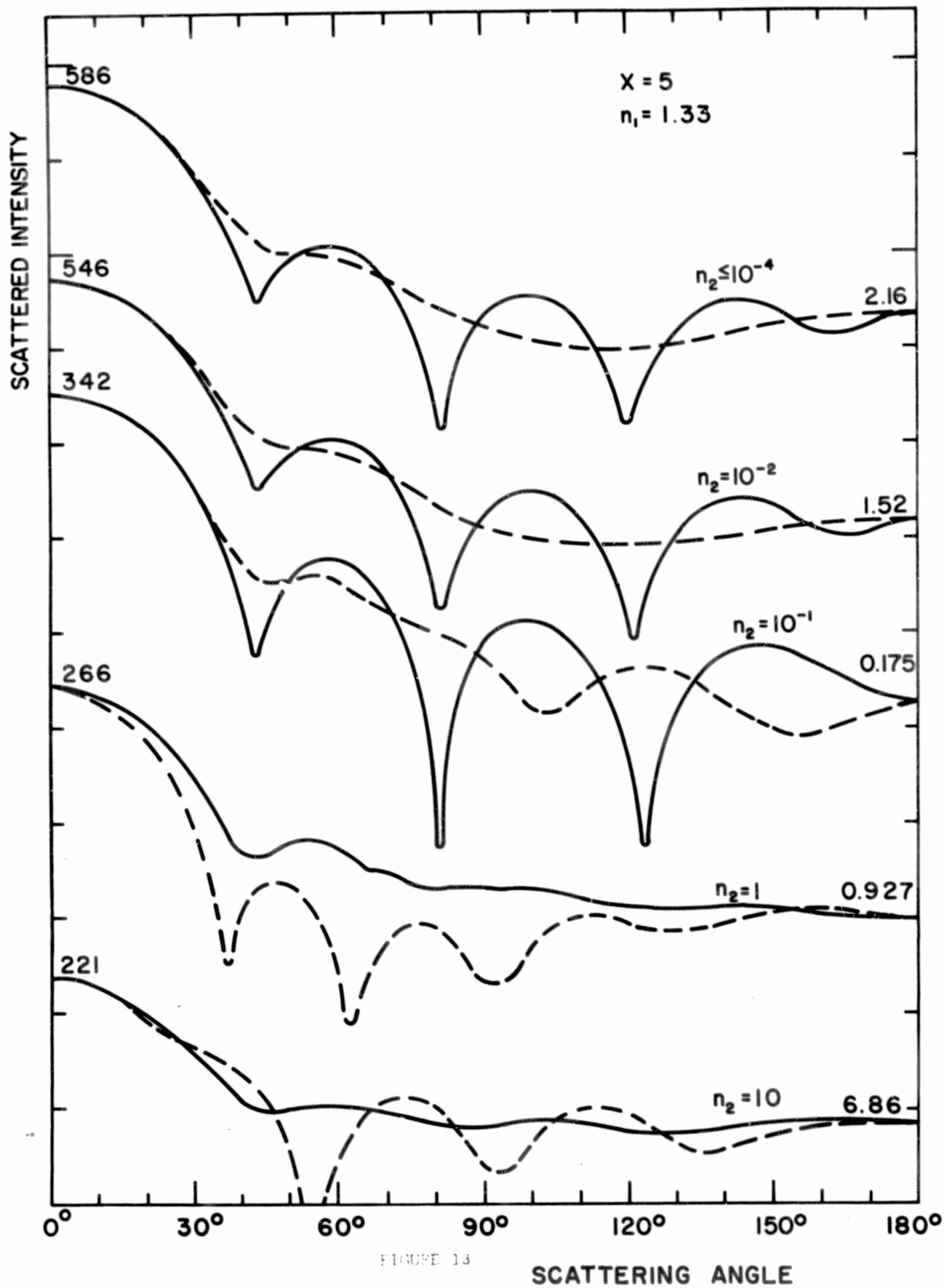


FIGURE 17



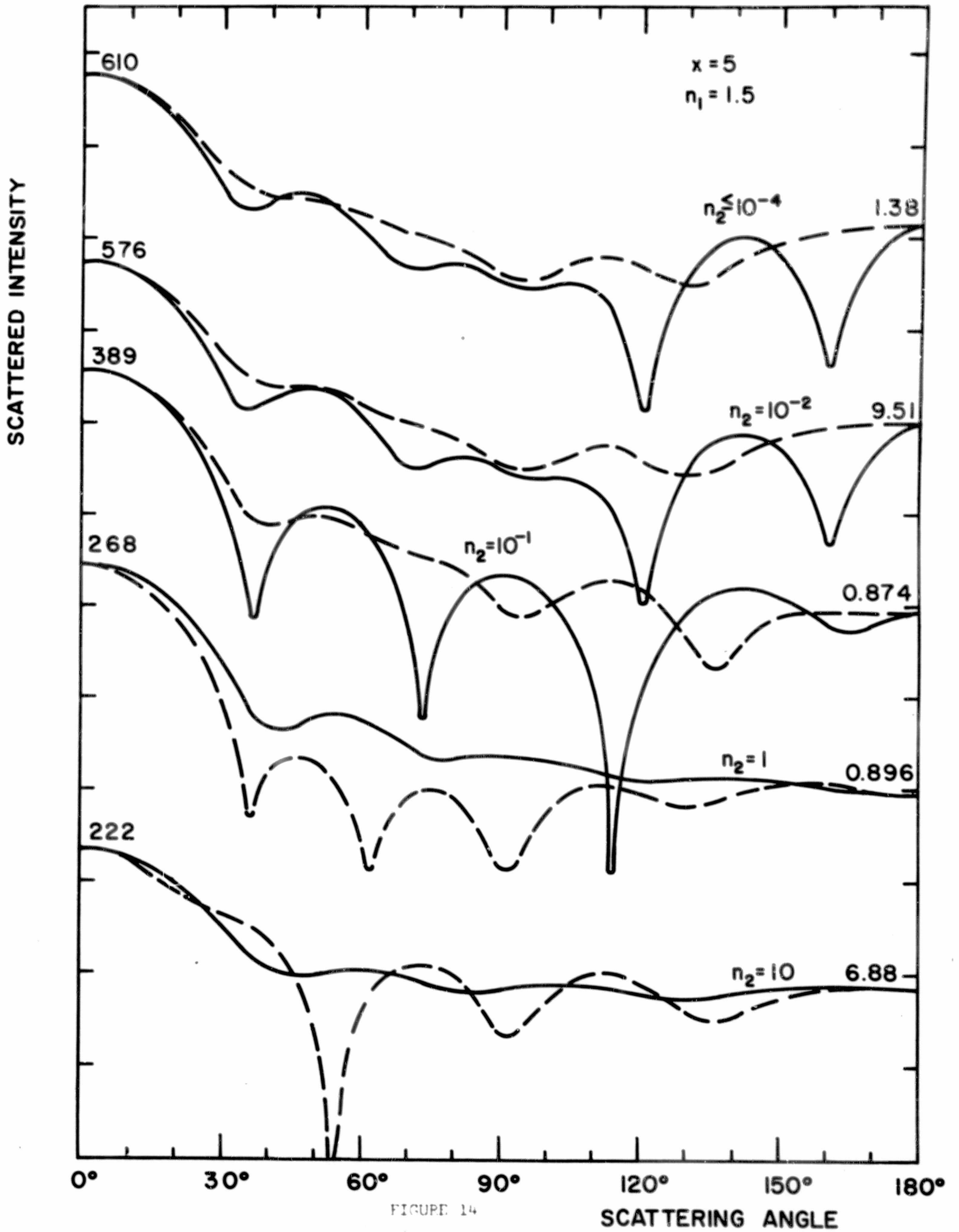


FIGURE 14

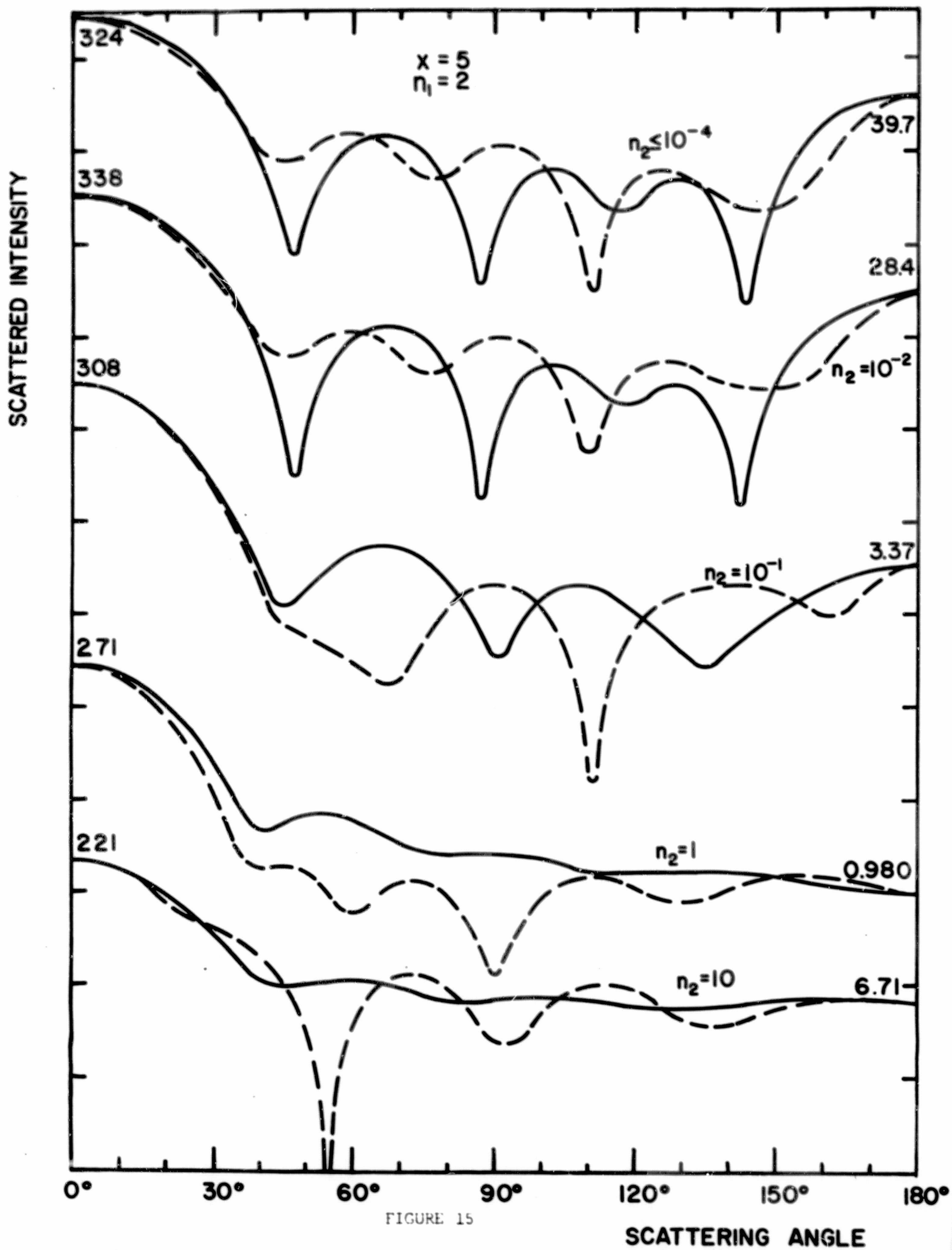


FIGURE 15

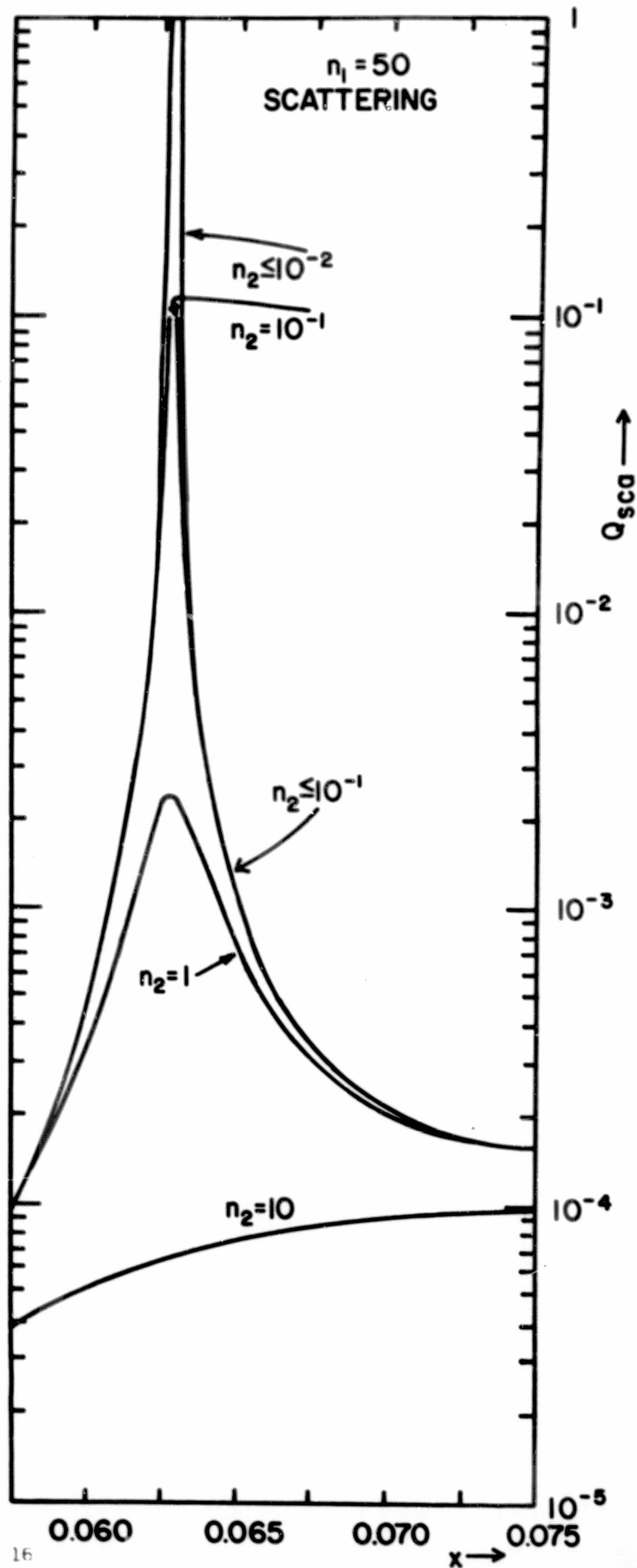
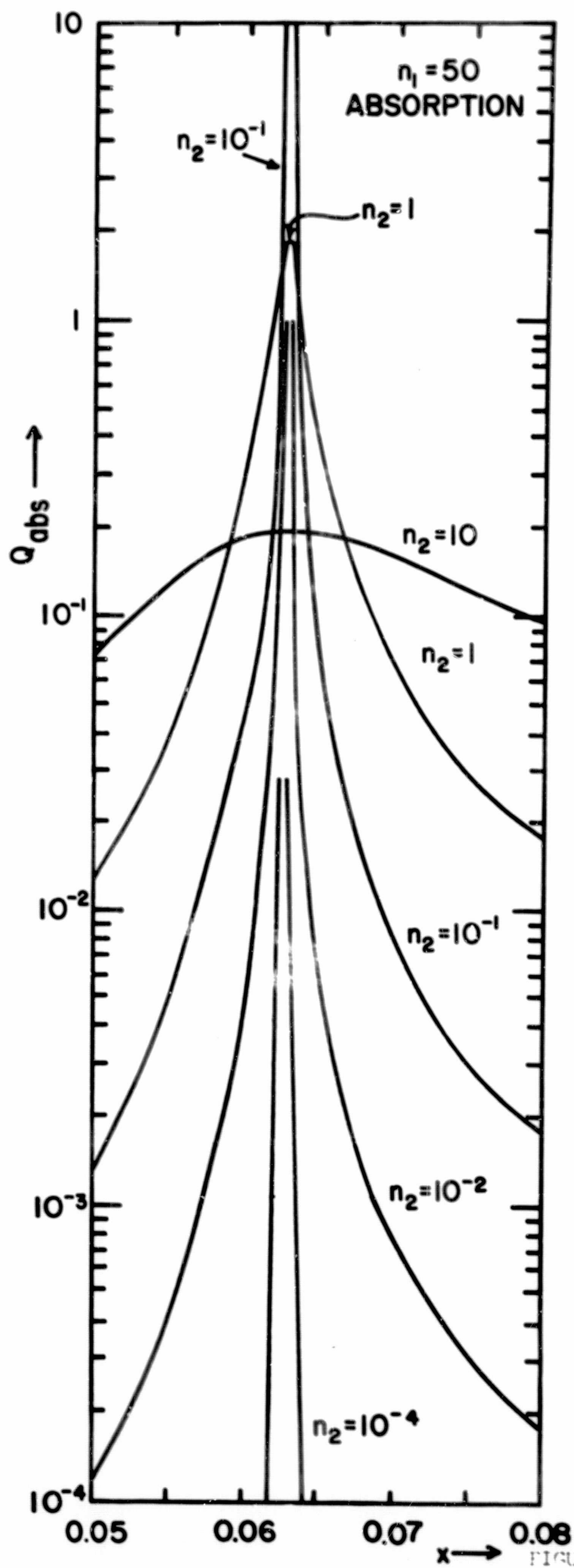


FIGURE 16